§19. Neoclassical Electron and Ion Transport in Toroidally Rotating Plasmas

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Neoclassical transport processes of electrons and ions are investigated in detail for toroidally rotating axisymmetric plasmas with large flow velocities on the order of the ion thermal speed [1]. The magnetic field and the toroidal flow velocity $\mathbf{V}_0 = \mathcal{O}(v_{Ti})$ are written as $\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla\Psi$ and $\mathbf{V}_0 = -Rc(\partial\Phi_0/\partial\Psi)\hat{\zeta}$, where ζ is the toroidal angle, Ψ represents the poloidal flux, and Φ_0 denotes the lowest-order electrostatic potential in the small-gyroradius expansion. For the rotating plasma consisting of electrons and single-species ions, the neoclassical transport equations are written as

$$\begin{bmatrix} \Gamma_e \\ q_e/T_e \\ q_i/T_i \\ \Pi_i \\ J_E \end{bmatrix} = \begin{bmatrix} L_{11}^{ee} & L_{12}^{ee} & L_{12}^{ei} & L_{1V}^{e} & L_{1E}^{e} \\ L_{21}^{ee} & L_{22}^{ee} & L_{22}^{ei} & L_{2V}^{e} & L_{2E}^{e} \\ L_{21}^{ie} & L_{22}^{ie} & L_{22}^{ii} & L_{2V}^{i} & L_{2E}^{i} \\ L_{V1}^{e} & L_{V2}^{e} & L_{V2}^{i} & L_{VV} & L_{VE} \\ L_{E1}^{e} & L_{E2}^{e} & L_{E2}^{i} & L_{EV} & L_{EE} \end{bmatrix} \begin{bmatrix} X_{e1}^{*} \\ X_{e2} \\ X_{i2} \\ X_V \\ X_E \end{bmatrix}.$$

Here, the neoclassical radial particle flux, electron and ion heat fluxes are denoted by $\Gamma_e(=\Gamma_i)$, q_e, and q_i, respectively. Π_i represents the neoclassical radial flux of the toroidal momentum, and $J_E \equiv \langle BJ_{\parallel}\rangle/\langle B^2\rangle^{1/2}$ is the averaged parallel current. The thermodynamic forces are given by $X_{e1}^{*} \equiv -N_e^{-1}\partial(N_eT_e)/\partial r - N_i^{-1}\partial(N_iT_i)/\partial r$, $X_{e2} \equiv -\partial T_e/\partial r$, $X_{i2} \equiv -\partial T_i/\partial r$ $X_V \equiv -\partial(V_0/R)/\partial r$, and $X_E \equiv \langle BE_{\parallel}\rangle/\langle B^2\rangle^{1/2}$.

The neoclassical transport coefficients L's depend on the toroidal flow velocity V_0. The Onsager relations for the flow-dependent neoclassical transport coefficients are derived from the symmetry properties of the drift kinetic equation with the self-adjoint collision operator. The complete neoclassical transport matrix with the Onsager symmetry is obtained for the Pfirsch-Schlüter and banana regimes.

The toroidal flow dependence of the ion thermal diffusivity $\chi_i = T_iL_{22}^{ii}/n_i$ is expressed in terms of the enhancement factor $F(\Upsilon)$ which is given for the banana regime as

$$\begin{aligned} F(\Upsilon) &\equiv \frac{\chi_i(V_0)}{\chi_i(V_0=0)} \\ &= 1 + 0.765\Upsilon - 0.631\Upsilon^2 + 0.280\Upsilon^3 \end{aligned}$$

where

$$\Upsilon \equiv \frac{m_iV_0^2}{(T_e+T_i)}.$$

This enhancement factor is in good agreement with that given by Catto *et al.* in spite of the difference between the solution methods.

The full expression of the parallel current for the toroidally rotating plasma in the banana regime is given by

$$\begin{aligned} J_E = &-\left(\frac{r}{R_0}\right)^{1/2}\frac{c}{B_P}\left[2.411\ F_{E1}^{e}(\Upsilon)\frac{dP}{dr}\right. \\ &\left. -1.800\ F_{E2}^{e}(\Upsilon)\,n_e\frac{dT_e}{dr} - 2.828\ F_{E2}^{i}(\Upsilon)\,n_i\frac{dT_i}{dr}\right] \\ &+\sigma_S\left[1 - 1.832\left(\frac{r}{R_0}\right)^{1/2}F_{EE}(\Upsilon)\right]X_E \end{aligned}$$

with the Spitzer conductivity σ_S and the enhancement factors

$$\begin{aligned} F_{E1}^{e}(\Upsilon) &= 1 + 0.868\Upsilon - 0.539\Upsilon^2 + 0.229\Upsilon^3 \\ F_{E2}^{e}(\Upsilon) &= 1 + 2.248\Upsilon - 1.661\Upsilon^2 + 0.727\Upsilon^3 \\ F_{E2}^{i}(\Upsilon) &= 1 + 1.494\Upsilon - 1.022\Upsilon^2 + 0.434\Upsilon^3 \\ F_{EE}(\Upsilon) &= 1 + 0.431\Upsilon - 0.184\Upsilon^2 + 0.072\Upsilon^3. \end{aligned}$$

The sum of the neoclassical and classical toroidal momentum diffusivity for the banana regime is written as $\chi_\phi = L_{VV}/(n_im_iR_0^2) + \frac{3}{5}\rho_i^2/\tau_i = (\frac{1}{10}q^2 + \frac{3}{5})\rho_i^2/\tau_i$. The banana inward-pinch velocity for the particles and the toroidal momentum due to the parallel electric field is derived from $v_{\text{inward}} = -L_{1E}^{e}X_E/n_e = -L_{VE}^{e}X_E/(n_im_iR_0V_0)$ as

$$v_{\text{inward}} = 2.411\ F_{E1}^{e}(\Upsilon)\left(\frac{r}{R_0}\right)^{1/2}\frac{cE_{\parallel}}{B_P}.$$

References
1) Sugama, H. and Horton, W. : Phys. Plasmas **4** (1997) 2215.